Motion-Induced and Parametric Excitations of Stay Cables: A Case Study

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ABSTRACT

This paper presents an overview of existing theoretical methods for the analysis of motion-induced and parametric excitations of stay-cables. Application of the methods is illustrated with a practical example of assessment of stay vibrations due to wind buffeting on the proposed Ironton-Russell Bridge, in Ohio. The results obtained from this analysis are compared against direct numerical (finite element) simulations. The selected scheme for cable vibration mitigation is presented, together with a discussion of its effectiveness.

INTRODUCTION

Among the many challenges facing wind engineers in the design of cable-supported bridges, is the assessment and mitigation of stay-cable vibrations. Stay-cables vibrate due to their inherently low damping, lightness, and flexibility, when excited by direct wind loads (buffeting and vortex shedding), as well as anchorage motions due to the bridge’s response to operational loads (wind, traffic, pedestrians etc.). Full-scale observations of cable-stayed bridge vibrations confirm that indirect, or ‘motion-induced’ vibrations can result in significant cable displacement amplitudes [Caetano, Macdonald]. These motions are a consequence of similarity between global modal frequencies of the bridge, and local modal frequencies of the cables.

In the case of a primary resonance, where there is a matching (or near-matching) of a global modal frequency of the bridge with the fundamental modal frequency of the cable, significant sway (out-of-plane) motions of the cable are possible. A second, and potentially more critical case, known as parametric resonance, can occur when a bridge modal frequency is twice the stay frequency. Theoretical results indicate that this can lead to large amplitude instabilities [Lilien and Pinto Da Costa], although this phenomenon has not been conclusively identified on any existing bridges.

Both primary and parametric resonance can result in displacements beyond those generally permitted for normal bridge operation, since they can cause problems associated with structural integrity and fatigue, or invoke discomfort in users of the bridge. It is therefore important that a detailed assessment of motion-induced vibrations
is conducted when designing a bridge, and there, if necessary, mitigation measures be implemented to prevent excessive cable motions.

In this paper, theoretical methods for the prediction of motion-induced stay-cable vibrations due to wind loading are reviewed. Application of these methods is illustrated with data from the actual cable study of the proposed Ironton-Russell Bridge in Ohio. A comparison between the analytical formulation and direct numerical simulations of the stay-cable motions is presented. The paper presents the cable vibration mitigation scheme selected for the Ironton-Russell Bridge and concludes with a discussion of its effectiveness at suppressing cable vibrations.

**Motion-Induced and Parametric Excitation of Cables**

Motion-induced cable vibrations, and cable-bridge interaction phenomena are highly complex, and as a result, generally not well understood. Recently there have been a number of notable developments in the literature [Macdonald, Georgakis, Caetano, Gattulli], focusing on explanation of some of the phenomena. However, the results from these resent studies have yet to lend themselves to a widespread practical implementation. This is due, in part, to the computational requirement for solution of the non-linear equation of motion, which typically restricts the analysis to a single cable.

In practice, simplifying assumptions to the non-linear cable behavior can be made, resulting in methods that are more easily applied to the assessment of multiple cables during the design [SETRA, Virolgeux]. In this paper, these simplified methods are adopted for the analysis of cables on the Ironton-Russell Bridge. Details of the selected methodology are provided below.

**Analysis Parameters**

Figure 1 illustrates the geometry of a moving stay-cable of length \( l \), subjected to average tension \( T \), anchorage motions \( \delta \), and an inclination angle \( \Theta \) with respect to the deck. In this paper, only the response of the cable to wind is considered, however, the

![Figure 1](image-url)
results could be extended to other excitations if estimates of the overall bridge responses are available.

Associated with the $m^{th}$ deck/tower mode is a set of anchorage modal displacements (estimated from a finite element model), which are transformed into a component along the chord axis ($X_m$), and a component perpendicular to the chord axis, in the plane of cable vibration ($Y_m$). These components are computed using the normalized mode shape coordinates for each anchorage ($\delta_x$, $\delta_y$, $\delta_z$, taken from the modal analysis), and the generalized amplitude coordinates computed from the buffeting analysis ($a_m$), as follows:

\[
X_m = \left[ x_m^t - x_m^b \right] a_m, \quad \text{and,} \quad Y_m = \left[ y_m^t - y_m^b \right] a_m, \tag{1}
\]

where the values $x_m$ and $y_m$ are the mode shape components along the chord, and perpendicular to the chord in the plane of cable motions, respectively. These values are computed via projections onto the chord axis, and onto the plane perpendicular to the chord axis, respectively. The superscripts “t” and “b” represent “top” (tower level) and “bottom” (deck level). The quantities $X_m$ and $Y_m$ are the net end displacements of the cable, scaled according to the generalized coordinate $a_m$, that could be either estimated theoretically via wind buffeting analysis, directly measured from an aeroelastic model test, or recorded from an existing bridge.

A key parameter for the quantification of the cable response is the ratio of the $m^{th}$ bridge (deck/tower) frequency ($\Omega_m$) to the fundamental (lowest) modal frequency of the cable ($\omega_1$, obtained from the linear free-vibration theory):

\[
r_m = \frac{\Omega_m}{\omega_1}, \tag{2}
\]

Motion-induced and parametric excitation amplitudes are most severe when the bridge modal frequency is equal to, or twice, the cable’s fundamental frequency (i.e., $r_m = 1$, or $r_m = 2$).

Since there is an uncertainty regarding the true values of the estimated frequencies, judgment should be applied in practice when computing (2). The authors are proposing as a conservative approach, to round values of $r_m$ close to 1.0 (or 2.0) within ±10% deviation. This is considered consistent with the expected accuracy of the frequency predictions.

The total damping of a stay ($\xi_T$) comprises aerodynamic ($\xi_A$) and structural damping ($\xi_S = c/2\mu\omega_1$):

\[
\xi_T = \xi_A + \xi_S = \left[ \frac{\beta \rho U D C_D + c}{2\mu \omega_1} \right], \tag{3}
\]

where $\rho$ is the density of air, $U$ is the mean wind speed for the analysis (corresponding to the value used for estimation of the generalized coordinates $a_m$), $D$ is the cable diameter, $C_D$ is the drag coefficient, $c$ is the structural viscous damping constant for the cable, and $\beta$ is a constant equal to 0.5 for vertical (in-plane) motion, and equal to 1 for sway (out-of-
plane) motion. Full-scale measurements of stay cable properties indicated very low structural damping values, $\xi_s \approx 0.05\%$ [Stoyanoff et al] which was retained for the current study.

**Motion-Induced Excitations due to Vertical and Lateral Anchorage Displacements**

In this paper, the methodology presented by Virlogeux is used for the estimation of resonant cable responses. A similar method was proposed by SETRA, however, based on direct numerical comparisons by the authors, it was determined that Virlogeux’s formulation provides more realistic estimates (particularly for the case of shorter stays). In this case, a harmonic excitation function of the following form is assumed for the $n^{th}$ cable:

$$y_{\nu}(t) = Y_{\nu m} \cos(\Omega_{\nu m} t).$$

Overlooking variations in tension and longitudinal displacements, the maximum response for each mode is given by:

$$A_{k, \nu m} = Y_{\nu m} \cdot \frac{2r_{mn}^2}{\pi k^3} \cdot H\left(\xi, \frac{r_{mn}}{k}\right),$$

where,

$$H\left(\xi, \frac{r_{mn}}{k}\right) = \frac{1}{\sqrt{\left(1 - \frac{r_{mn}^2}{k^2}\right)^2 + \left(\frac{2\xi}{r_{mn}^2} \cdot \frac{r_{mn}}{k}\right)^2}},$$

is the mechanical admittance function.

It is clear from (6) that the displacement is a maximum when $r_{mn}$ is unity. The total response of the $n^{th}$ cable for the $m^{th}$ bridge mode is given by the summation:

$$A_{m} = \sqrt{\sum_{k=1}^{K} A_{k, \nu m}^2},$$

where $K$ is the number of cable modes considered ($K = 10$ in this study) The total response of the $n^{th}$ cable for all $M$ bridge (deck/tower) modes considered in the analysis is then calculated as:

$$A_{n} = \sqrt{\sum_{m=1}^{M} A_{mn}^2}.$$
Parametric Excitation by Along-the-Cable-Chord Anchorage Displacements

Considering now an along-the-cable-chord harmonic excitation of the cable of the following form:

\[ x_{mn}(t) = X_m \cos(\Omega_m t) , \]  

and recognizing that the variation in cable tension \( T \) can no longer be neglected, results in the non-linear Mathieu-Hill equation [Nayfeh and Mook] governing the transverse response \( y(t) \):

\[
\ddot{y}(t) + 2\omega_i \xi \dot{y}(t) + \omega_i^2 \left[ 1 + \frac{ES_n X_m}{Tl} \cos(\Omega_m t) \right] y(t) + 3\pi \frac{ES_n g}{Tl} \cos(\Theta) y^2(t) + \frac{\pi^2}{4} \frac{ES_n}{T} \omega_i^2 \cos(\Omega_m t) = 0 ,
\]

where \( S_n \) is the cross-sectional area of the \( n^{th} \) cable, and \( E \) is Young’s Modulus. The second coefficient of \( y(t) \) is the so-called parametric excitation term, due to the presence of the input excitation function.

Two zones of instability govern the response of a cable under this excitation regime. Zone 1 corresponds to \( r_{mn} = 2 \), and Zone 2 corresponds to \( r_{mn} = 1 \). The limit amplitudes for each region are given by [SETRA]:

\[
A_{mn}^{\text{Zone 1}} = 2 \omega_i l \left[ \left( \frac{r_{mn}}{2} \right)^2 - 1 + \sqrt{\left( \frac{ES_n X_m}{2Tl} \right)^2 - \left( \frac{\xi \Omega_m}{\omega_i r_{mn}} \right)^2} \right] ,
\]

and

\[
A_{mn}^{\text{Zone 2}} = \frac{-2 ES_n X_m g \cos(\Theta)}{\pi T l \omega_i^2 \left( 1 - \frac{2}{r_{mn}^2} - \frac{1}{2} \left( \frac{ES_n X_m}{Tl} \right)^2 \right)} .
\]

APPLICATION TO THE PROPOSED IRONTON-RUSSELL BRIDGE

The methods presented in the previous sections were applied to the stay cables of the proposed Ironton-Russell Bridge over the Ohio River. This bridge consists of a single tower and a two-deck with 34 back span, and 36 main span stays. The bridge is approximately symmetric about the center of the deck. The length of the longest stay is approximately 960 ft, and estimated fundamental stay frequencies vary between 0.44 Hz and 1.6 Hz.
Early in the design, a preliminary assessment of the cables indicated the possibility for motion-induced and parametric excitations, due to the proximity of several deck/tower (global) modal frequencies, to that of the stay cables (Figure 2).

As indicated in Figure 2, several deck/tower modes lay in close proximity to once, and twice- the stay frequencies (i.e., \( r_{mn} \approx 1 \) and \( r_{mn} \approx 2 \)). A particular concern during the assessment was the frequency proximity of bridge mode 2 (0.447 Hz lateral tower mode) to several of the frequencies of the longest stays. Since this was a lateral tower mode, it was recognized that mitigation could not be accomplished through the use of crossties, and, if necessary, a supplementary damper solution would be required.

**Parametric Excitation Assessment**

Figure 3 is a graphical illustration of the assessment of the potential for parametric instabilities to develop in cables R/L07 of the bridge, during the design wind speed storm of 69 mph. The reduced amplitude of the cable \( (a = ESX_m/2T) \) is plotted for 30-deck/tower modal anchorage displacements (corresponding to the first 30 modes of the deck/tower), together with the instability regions computed using the total damping \( (\xi_T) \) for the cable. Zone 2 is much broader than Zone 1, and requires smaller reduced amplitudes to be reached. However, the failure of any of the reduced amplitudes to reach the instability regions indicates that parametric excitation instability is unlikely to occur for this cable.
The assessment was conducted for all 35 cables on one of the cable planes, and it was determined that parametric instabilities are unlikely to occur on the Ironton-Russell Bridge. It should be noted that for the bridge modes higher that about 0.7 Hz, the predicted buffeting responses were very low.

**Motion-Induced Stay Vibration Amplitudes**

The motion-induced response amplitudes of each cable were estimated for the design wind speed using (6) through (9). The damping for each cable was estimated using (4), the value of $\beta$ was selected according to the type of motion induced in the stay, by the deck/tower (i.e., in-plane or out-of-plane). Figure 4 plots the predicted maximum displacement amplitude of each cable at both the design wind speed of 69 mph, and the service wind speed 46 mph. The values shown are with respect to the equilibrium position of the cable (i.e., multiply by two to arrive at peak-to-peak amplitudes). Also shown on the plot is the acceptable amplitude criterion applied to each cable, which was selected as $0.5D$, or half of the cable diameter ($1D$ peak-to-peak), at the service wind speed. The amplitude was selected based on concerns for human comfort on the bridge, and has been previously adopted on other bridge projects [Bosch]. Except for six of the shortest cables, the peak amplitudes exceed the recommended amplitude criterion for both the design and service wind events.

Table 1 lists the maximum modal contributions ($A_{mn}$) to the total response of cables R/L01, R/L02, R/L33, R/L34, and R/L35, for the design wind speed. Modes 2 (lateral tower motion) and 3 (vertical deck motion) of the deck/tower were found to be the primary contributors to the response of these cables.
FIGURE 4
PREDICTED MAXIMUM CABLE RESPONSE AMPLITUDES FROM LATERAL ANCHORAGE DISPLACEMENTS.

<table>
<thead>
<tr>
<th>Cable</th>
<th>Peak total response (ft)</th>
<th>Maximum modal response (ft)</th>
<th>Bridge mode causing max response</th>
</tr>
</thead>
<tbody>
<tr>
<td>R/L01</td>
<td>2.71</td>
<td>2.69</td>
<td>2 – 1st Lateral tower mode</td>
</tr>
<tr>
<td>R/L02</td>
<td>3.16</td>
<td>2.66 1.66</td>
<td>2 – 1st Lateral tower mode 3 – 2nd vertical mode</td>
</tr>
<tr>
<td>R/L33</td>
<td>4.36</td>
<td>3.36 2.52</td>
<td>3 – 2nd vertical mode 2 – 1st Lateral tower mode</td>
</tr>
<tr>
<td>R/L34</td>
<td>3.79</td>
<td>2.68 2.5</td>
<td>3 – 2nd vertical mode 2 – 1st Lateral tower mode</td>
</tr>
<tr>
<td>R/L35</td>
<td>3.01</td>
<td>2.36 1.76</td>
<td>3 – 2nd vertical mode 2 – 1st Lateral tower mode</td>
</tr>
</tbody>
</table>

TABLE 1
PREDICTED MAXIMUM MODAL RESPONSES OF TEN OF THE LONGEST CABLES ON THE BRIDGE (69 MPH WINDS).

Comparison with Direct Numerical Simulations

A finite element model of cable R35 was developed using the SAP2000 Non-linear software, and compared with the analytical results. The tension of the cable was simulated using initial strains, and the non-linear geometric stiffness matrix was evaluated for estimation of eigenvalues. A sinusoidal excitation function with a frequency of 0.44 Hz was used as input to the anchorages. The simulation predicted a peak cable displacement of 2.4 ft, which was almost exactly that calculated using the analytical method (see Table 1). The numerical approach was retained for further analysis of vibration mitigation schemas.
MITIGATION OF STAY CABLE VIBRATIONS

Based on the motion-induced and parametric excitation vibration assessments, it was proposed a mitigation scheme consisting of both cable crossties, and external lateral damping devices, required to reduce the anticipated cable vibrations to the required level of $0.5D$, at the service wind speed 46 mph.

Cross-Tie System

The accepted crosstie schema will increase vertical stiffness of the back span and main span cable fans, connecting all cables with the deck. Pretension of the ties was designed to sustain the maximum cable dynamic displacements, avoiding any slackening, which could result in failures of the ties during extreme wind events.

A finite element model of the main span and back span cable fans was developed using the SAP2000 Non-linear software. Figure 5 shows the first in-plane mode shapes for the tied back span and main span cable arrays. Table 2 lists a comparison between the in-plane and out-of-plane fundamental modal frequencies of the un-tied and tied cable arrays.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Main Span Array</th>
<th>Back Span Array</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Un-tied frequency (Hz)</td>
<td>Tied frequency (Hz)</td>
</tr>
<tr>
<td>First out-of-plane</td>
<td>0.440</td>
<td>0.460</td>
</tr>
<tr>
<td>First in-plane</td>
<td>0.441</td>
<td>1.117</td>
</tr>
</tbody>
</table>

The installation of crossties can have a significant affect on the in-plane modal frequencies of the system, whereas, the affect on out-of-plane frequencies is negligible. As a result, the crossties could not be expected to reduce out-of-plane motions due to lateral motions of the tower, and supplemental dampers were deemed necessary on the five longest cables with fundamental frequencies in the range of 0.44-0.46 Hz.
External Lateral Dampers

The external lateral damping devices (ELDs) were designed to provide the maximum possible amount of damping to the cables, given the constraints on their placement along the deck. Figure 6 shows graphics of Motioneering’s design of the ELDs. Five of the devices are proposed for each side of the deck, on the cables listed in Table 1. The ELD units are expected to contribute approximately 4% additional equivalent modal damping for the first mode of each of these cables.

FIGURE 6
PROPOSED EXTERNAL LAT ERAL DAMPING DEVICE FOR SUPPRESSION OF OUT-OF-PLANE CABLE VIBRATIONS.

Effectiveness of Proposed Mitigation Scheme

An equivalent finite element model of cable longest cable on the main span (R35) was developed consisting of spring elements representing the crossties, and a dashpot representing the external lateral damper. The first and second out-of-plane, and first in-plane frequencies of the reduced system were tuned to within 2% of the full fan model (Figure 5). A 40-minute time history of wind buffeting loads and responses [Stoyanoff], estimated from a simulation of the design windstorm, was used as input to the cables and their anchorages. Simulations of the reduced model were run for three scenarios: 1) no mitigation, 2) ELD only (no cross ties), 3) ELD and crossties. Results from the simulations are shown in Table 3.

<table>
<thead>
<tr>
<th>Response</th>
<th>Predicted peak displacement: unmitigated (ft)</th>
<th>Predicted peak displacement: External Lateral Dampers only (ft)</th>
<th>Predicted peak displacement: External Lateral Dampers and Crossties (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral</td>
<td>2.58</td>
<td>0.74</td>
<td>0.73</td>
</tr>
<tr>
<td>Vertical</td>
<td>1.11</td>
<td>1.11</td>
<td>0.08</td>
</tr>
<tr>
<td>Total</td>
<td>2.86</td>
<td>1.30</td>
<td>0.73</td>
</tr>
</tbody>
</table>

TABLE 3
PREDICTED CABLE DISPLACEMENTS OF CABLE R35 (69 MPH WINDS).
The ELDs are expected to reduce out-of-plane displacements to approximately one-quarter, and the crossties are expected to decrease in-plane displacements to at least one-tenth of their un-mitigated amplitudes. Considering that the cable pipe diameter is approximately 9 in, the resulting amplitude would be reduced from approximately $4D$ to $1D$, for the design windstorm.

For the serviceability windstorm (46 mph), maximum cable displacements were estimated to be approximately 45 – 50% of the values listed in Table 3 (i.e, peak displacements of approximately 0.32 – 0.37 ft). These values are less than, or approximately equal to, the $0.5D$ criterion. Since it was recognized that the serviceability wind speed will occur more frequently, and that the bridge would likely be closed for service during the design wind speed event, the proposed mitigation schema was deemed acceptable for the design.

**CONCLUSIONS**

Analytical techniques for estimation of motion-induced and parametric excitation of stay cables were presented, and applied to the proposed Ironton-Russell Bridge over Ohio River. The analytical methods predictions were in a good agreement with these obtained via direct numerical simulations on the longest cables of the bridge.

Results from the cable vibration assessment indicated the requirement for a vibration mitigation scheme consisting of both crosstie cables and external lateral damping devices. These devices were designed, and their performance verified, with numerical simulations of the main span and back span cable arrays. Simulation results indicated significant reductions in in-plane and out-of-plane displacements of the longest cables exited by bridge buffeting. For the proposed mitigation scheme, the maximum cable displacements during serviceability wind events are expected to be less than one cable diameter, peak-to-peak.

**ACKNOWLEDGEMENTS**

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**REFERENCES**


